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INTERACTING STRATEGY SETS IN MULTIOBJECTIVE COMPETITION; A DOMINANCE CONE CONSTRAINED GAME SOLUTION

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FOURTH INTERNATIONAL CONFERENCE ON THE FOUNDATIONS AND APPLICATIONS OF UTILITY, RISK AND DECISION THEORY

ABSTRACT

Interacting Strategy Sets in Multiobjective Competition; A Dominance Cone Constrained Game Solution

by

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(The University of Texas at Austin)

Models employed in evaluation or stipulation of regulatory policies involving competition and/or differing objectives of competing parties have sometimes been seriously deficient in accounting for interactions between the parties' strategies or in allowing for multiple objectives. Classical game-theoretical models, wherein the strategy set is the topological product of the individual parties strategy sets, do not encompass such situations. The new "dominance cones" method and class of solutions (Charnes, Cooper, Huang, Wei) is herein further extended to such extensions of classical games. This is applied to an example of Harker without requiring his variational and quasi-variational inequalities or point-to-set mappings.

KEY WORDS

Dominance Cones

Multipayoff Cross-Constrained Games

Variational Inequalities

INTERACTING STRATEGY SETS IN MULTIOBJECTIVE COMPETITION; A DOMINANCE CONE CONSTRAINED GAME SOLUTION

by

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1. Introduction

Models employed in evaluation or stipulation of regulatory policies involving competition and/or differing objectives of competing parties have sometimes been seriously deficient in accounting for interations between the parties' strategies or in allowing for multiple objectives. Classical game-theoretical models, wherein the strategy set is the topological product of the individual parties stragy sets, do not treat such situations. They can however be handled by the new "dominance cones" method and class of solutions (Charnes, Cooper, Wei and Huang [2]) to such extensions of classical games. Herein we extend the "C^2WH" method to obtain more general results looking forward to resolution (in a later paper) of a competitive situation in transportation with interacting strategy sets [4].

Harker (1986) [5] used the Variational Inequality (VI) method to discuss the Generalized Nash Equilibrium Games (GNE) and gave an example in which only one solution can be found by (VI). Using the "dominance cones" method given in our paper, we can also find all the (GNE) solutions of the example given by Harker.

2. Nondominated Equilibrium Points

Definition 2.1: Let S be a set in E^m, the set $S^* = \{y \in E^m : x^T y \le 0$ for all $x \in S\}$ is called the negative polar cone of S.

Definition 2.2: Let Λ be a cone in E^m. Λ is said to be "acute" if there exists an open half-space

$$H = \{x \in E^m : a^T x > 0\ ,\ a \neq 0\}$$

such that

$$\bar{\Lambda} \subset H \cup \{0\}$$

Lemma 2.1: Let Λ and Λ_1 be cones in E^m

(i) If $\Lambda \subset \Lambda_1$, then $\Lambda^* \supset \Lambda_1^*$.

(ii) Int $\Lambda^* \neq \phi$ if and only if Λ is acute.

(iii) When Λ is acute, Int $\Lambda^* = \{y \in E^m : x^T y < 0$ for all $x \in \overline{\Lambda}$ and $x \neq 0\}$ and

$\overline{\Lambda} \cap (-\Lambda) = \{0\}$.

(iv) If Λ is a convex cone, then $(\Lambda^*)^* = \overline{\Lambda}$.

Definition 2.3: Let S be a set in E^m, $\overline{x} \in \overline{S}$.

The tangency cone of S at $\overline{x}$ is denoted by $T(S, \overline{x})$:

$$T(S, \overline{x}) = \Big\{ h \in E^m : \text{there exists a sequence } \{x^k\} \text{ and a sequence } \{\lambda^k\} \text{ such that}$$

$$h = \lim_{k \to \infty} \lambda_k (x^k - \overline{x}), \text{ where } x^k \in S,\ \lambda_k > 0 \text{ and } \lim_{k \to \infty} x_k = \overline{x} \Big\}$$

For definitions and properties of cones, polar cones and direction cones the reader is referred to [1], [3], [6], [7], [8] and [9].

Definition 2.4: Let S be a convex set in E^m, Λ be a convex cone in E^n. A real-valued vector function $G : S \to E^n$ is Λ-concave on S if

$$G\left(\lambda x^1 + (1-\lambda) x^2\right) - \left(\lambda G(x^1) + (1-\lambda) G(x^2)\right) \in \Lambda$$

for all $x^1, x^2 \in S$ and $\lambda \in (0,1)$.

Lemma 2.2: Let Λ be a closed convex cone in E^n, S be a convex set in E^m, $G : S \to E^n$ be differentiable in a open set which contains S. If G is Λ-concave on S, then for every $x^1, x^2 \in S$, we have

$$G(x^1) + \nabla_x G(x^1)(x^2 - x^1) \in G(x^2) + \Lambda$$

Proof: Since G is Λ-concave on S, for every $x^1, x^2 \in S$

$$G\left(\lambda x^2 + (1-\lambda) x^1\right) - \left(\lambda G(x^2) + (1-\lambda) G(x^1)\right) \in \Lambda \text{ for all } \lambda \in (0,1)$$

Thus

$$G(x^1) + \left[G\left(x^1 + \lambda(x^2 - x^1)\right) - G(x^1)\right] / \lambda \in G(x^2) + \Lambda$$

Letting $\lambda \to 0^+$, we have

$$G(x^1) + \nabla_x G(x^1)(x^2 - x^1) \in G(x^2) + \Lambda$$

Q.E.D.

<u>Lemma 2.3</u>: Let S be a convex set in E^m, Λ be a convex cone in E^n, $G : S \rightarrow E^n$. If G is Λ - concave on S, then for every $p \in (-\Lambda^*)$, $P^T G$ is concave on S.

<u>Proof</u>: Since G is Λ - concave on S, for all $x^1, x^2 \in S$ and all $\lambda \in (0,1)$, we have

$$G\left(\lambda x^1 + (1-\lambda)x^2\right) - \left(\lambda G\left(x^1\right) + (1-\lambda)\, G\left(x^2\right)\right) \in \Lambda$$

Thus for every $p \in (-\Lambda^*)$, we have

$$p^T G\left(\lambda x^1 + (1-\lambda)x^2\right) \geq \lambda p^T G\left(x^1\right) + (1-\lambda) p^T G\left(x^2\right)$$

for all $\lambda \in (0,1)$.

Q.E.D.

<u>Lemma 2.4</u>: Let S be a convex set in E^m, Λ be a closed convex cone in E^n, $G : S \rightarrow E^n$. If, for arbitrary $p \in \Lambda^*$, $P^T G$ is concave on S, then G is $(-\Lambda)$ - concave on S.

<u>Proof</u>: Since for arbitrary $p \in \Lambda^*$ and $x, y \in S$, we have

$$p^T G\left(\lambda x + (1-\lambda)y\right) \geq \lambda p^T G(x) + (1-\lambda) p^T G(y) \text{ for all } \lambda \in (0,1)$$

i.e.,

$$p^T \left\{ G\left(\lambda x + (1-\lambda)y\right) - \left(\lambda G(x) + (1-\lambda) G(y)\right) \right\} \geq 0 \quad \text{for all } \lambda \in (0,1)$$

By Lemma 2.1,

$$G\left(\lambda x + (1-\lambda)y\right) - \left(\lambda G(x) + (1-\lambda) G(y)\right) \in -\left(\Lambda^*\right)^* = -\Lambda$$

for all $\lambda \in (0,1)$.

This means that G is $(-\Lambda)$ - concave on S.

Q.E.D.

A Multi-Payoff Constrained N-person Game in Normal Form

<u>Definition 2.5</u>: A multi-payoff constrained n-person game in normal form is given by n nonempty sets S_i in E^n (i = 1, 2, ..., n), the strategy sets of the players 1, 2, ..., n; a real-valued vector function $G = (g_1, ..., g_m)^T : S_1 x \ldots x S_n \rightarrow E^m$, the cross-constraint function of the n players; for each player i there is a real-valued vector function $A^i = \left(A^i_1, \ldots, A^i_\ell\right)^T : S_1 x \ldots x S_n \rightarrow E^\ell$, the vector payoff function of the ith player (i = 1, 2, ..., n); a convex cone K in E^m, the constraint cone; a convex cone W in E^ℓ, the dominance cone; $\Re_K = \left\{ x = (x_1, \ldots, x_n) : G(x) \in K,\ x_i \in S_i,\ i = 1, 2, \ldots, n \right\}$, the constraint set.

Such a game will be denoted by

$$\Gamma = \left\{\Re_K,\ W;\ A^1, \ldots, A^n\right\}.$$

Definition 2.6: Let $\Gamma = \left\{\Re_K,\ W;\ A^1, \ldots, A^n\right\}$ be a multi-payoff constrained n-person game.

$\bar{x} = (\bar{x}_1, \ldots, \bar{x}_n) \in \Re_K$ is called a nondominated equilibrium point of the game Γ associated with

W if there exists no $x = (x_1, \ldots, x_n) \in S_1 \times \ldots \times S_n$ satisfying

$(\bar{x}_1, \ldots, \bar{x}_{i-1}, x_i, \bar{x}_{i+1}, \ldots, \bar{x}_n) \in \Re_K$ $(i = 1, 2, \ldots, n)$ such that

$$A^i(\bar{x}_1, \ldots, \bar{x}_n) \in A^i(\bar{x}_1, \ldots, \bar{x}_{i-1}, x_i, \bar{x}_{i+1}, \ldots, \bar{x}_n) + W$$

and

$$A^i(\bar{x}_1, \ldots, \bar{x}_n) \neq A^i(\bar{x}_1, \ldots, \bar{x}_{i-1}, x_i, \bar{x}_{i+1}, \ldots, \bar{x}_n)$$

for all $i = 1, \ldots, n$.

Definition 2.7: Let $\Gamma = \left\{\Re_K,\ W;\ A^1, \ldots, A^n\right\}$ be a multi-payoff constrained n-person game. Then Γ is called a (W-K)-concave game if the following four conditions hold for all $i = 1, 2, \ldots, n$:

(i) S_i is a convex set in E^{n_i};

(ii) $A^i(x_1, \ldots, x_n)$ is $(-W)$-concave with respect to x_i on S_i for fixed $x_j \in S_j$ $(j \neq i, j = 1, 2, \ldots, n)$;

(iii) $A^i(x_1, \ldots, x_n)$ is continuous on $S_1 \times \ldots \times S_n$.

(iv) $G(x_1, \ldots, x_n)$ is continuous and K-concave on $S_1 \times \ldots \times S_n$.

In the remaining sections, except where specifically noted, we shall always use the following symbols:

(1) $W_n = \underbrace{W \times \ldots \times W}_{n}$, $W_n^* = \underbrace{W^* \times \ldots \times W^*}_{n}$, $K_n = \underbrace{K \times \ldots \times K}_{n}$

$K_n^* = K^* \times \ldots \times K^*$ and $S = S_1 \times \ldots \times S_n$;

(2) "Strictly nonzero" $p \in W_n^*$ implies $p = (p^1, \ldots, p^n)$, $p^i \in W^*$ and $p^i \neq 0$ for all $i = 1, 2, \ldots, n$;

(3) $Q \in -K_n^*$ implies $Q = (q^1, \ldots, q^n)$, $q^i \in -K^*$, $i = 1, 2, \ldots, n$;

(4) $x \in S$ implies $x = (x_1, \ldots, x_n)$ and $x_i \in S_i$ for all $i = 1, 2, \ldots, n$;

(5) For fixed $x \in \Re_K$, let for all $i = 1, 2, ..., n$

$$D_i(\bar{x}) = \left\{ x_i \in S_i : \left(\bar{x}_1, ..., \bar{x}_{i-1}, x_i, \bar{x}_{i+1}, ..., \bar{x}_n\right) \in \Re_K \right\}$$

and let $D(\bar{x}) \overset{\Delta}{=} D_1(\bar{x}) \times ... \times D_n(\bar{x})$.

Clearly, for all $i = 1, 2, ..., n$, we have $D_i(\bar{x}) \subset S_i$ and $D(\bar{x}) \subset S$.

(6) $x \in D(\bar{x})$ implies for $x = (x_1, ..., x_n)$ that $x_i \in D_i(\bar{x})$ for all $i = 1, 2, ..., n$.

(7) For a real-valued vector function $G(x) = (g_1(x), ..., g_m(x))^T$, we denote the "gradient" of $G(x)$ (really, the vector of gradients of G's n component) by

$$\nabla_{x_i} G(x) = \begin{pmatrix} \nabla_{x_i} g_1(x) \\ \vdots \\ \vdots \\ \nabla_{x_i} g_m(x) \end{pmatrix}, \quad (x_i \text{ is a vector!})$$

(8) For a specific $p \in W_n^*$, let

$$\Psi_P(x,y) = \sum_{i=1}^{n} p^{i^T} A^i \left(y_1, ..., y_{i-1}, x_i, y_{i+1} ..., y_n\right) \text{ for all } (x, y) \in S \times S.$$

(9) For a specific $p \in W_n^*$, the generalized Lagrangean function is defined as follows:

$$\Phi_P(x,y,Q) = \sum_{i=1}^{n} p^{i^T} A^i \left(y_1, ..., y_{i-1}, x_i, y_{i+1} ..., y_n\right) + \sum_{i=1}^{n} q^{i^T} G\left(y_1, ..., y_{i-1}, x_i, y_{i+1} ..., y_n\right)$$

for all $(x, y) \in S \times S$ and $Q \in -K_n^*$.

3. Computing Nondominated Equilibrium Points

In this section we outline a more general technique than was given previously in [2], which enables us to capture additional nondominated equilibrium points.

Lemma 3.1: Let $\Gamma = \left\{\Re_K, W; A^1, ..., A^n\right\}$. be a (W-K)-concave game. If $\bar{x} \in \Re_K$ is a nondominated equilibrium point of Γ associated with W, then there exists strictly nonzero $p \in W_n^*$ such that for arbitrary $x \in D(\bar{x})$, we have

$$p^{i^T} A^i\left(\bar{x}_1, ..., \bar{x}_{i-1}, x_i, \bar{x}_{i+1} ..., \bar{x}_n\right) \le p^{i^T} A^i(\bar{x}) \text{ for all } i = 1, 2, ..., n.$$

Proof: Suppose that $\bar{x} \in \Re_K$ is a nondominated equilibrium point of Γ associated with W. This means that there exist no n nonzero $w_i \in W$ (i = 1, 2, ..., n) and $x \in D(\bar{x})$ such that

$$A^i(\bar{x}) = A^i(\bar{x}_1, \ldots, \bar{x}_{i-1}, x_i, \bar{x}_{i+1} \ldots, \bar{x}_n) + w_i \text{ for all } i = 1, 2, \ldots, n.$$

For each i = 1, 2, ..., n, consider

$$\Lambda_i = \left\{ z \in E^l : z - A^i(\bar{x}_1, \ldots, \bar{x}_{i-1}, x_i, \bar{x}_{i+1} \ldots, \bar{x}_n) + A^i(\bar{x}) = w_i \right.$$
$$\left. \text{for some } x_i \in D_i(\bar{x}) \text{ and nonzero } w_i \in W \right\}$$

It is easy to show that Λ_i is a convex set and $0 \notin \Lambda_i$. Hence by the separation theorem, there exists nonzero $p^i \in E^l$ such that

$$xp^{i^T} z \le 0 \qquad \text{for all } z \in \Lambda_i.$$

Next, for arbitrary $x_i \in D_i(\bar{x})$, nonzero $w_i \in W$ and $\lambda > 0$, let

$$z_{x_i\lambda w_i} = A^i(\bar{x}_1, \ldots, \bar{x}_{i-1}, x_i, \bar{x}_{i+1} \ldots, \bar{x}_n) - A^i(\bar{x}) + \lambda w_i$$

Then $z_{x_i\lambda w_i} \in \Lambda_i$ and

$$p^{i^T} A^i(\bar{x}_1, \ldots, \bar{x}_{i-1}, x_i, \bar{x}_{i+1} \ldots, \bar{x}_n) - p^{i^T} A^i(\bar{x}) + \lambda p^{i^T} w_i \le 0$$

Thus

$p^i \in W^*$. Letting $\lambda \to 0^+$, we obtain

$$p^{i^T} A^i(\bar{x}_1, \ldots, \bar{x}_{i-1}, x_i, \bar{x}_{i+1} \ldots, \bar{x}_n) \le p^{i^T} A^i(\bar{x}) \quad \text{for all } i = 1, 2, \ldots, n.$$

Q.E.D.

Lemma 3.2: Let $\Gamma = \left\{\Re_K, W; A^1, \ldots, A^n\right\}$. , $\bar{x} \in \Re_K$ and $p \in \operatorname{Int} W_n^*$ if for arbitrary $x \in D(\bar{x})$, we have

$$p^{i^T} A^i(\bar{x}_1, \ldots, \bar{x}_{i-1}, x_i, \bar{x}_{i+1} \ldots, \bar{x}_n) \le p^{i^T} A^i(\bar{x}) \tag{3.1}$$

for all i = 1, 2, ..., n, then $\bar{x}$ is a nondominated equilibrium point of Γ associated with W.

Proof: Suppose to the contrary that $\bar{x}$ is not a nondomonated equilibrium point of Γ associated with W. That is, there exist $x \in D(\bar{x})$ and $w_i \in W$ such that

$$A^i(\bar{x}) = A^i(\bar{x}_1, \ldots, \bar{x}_{i-1}, x_i, \bar{x}_{i+1} \ldots, \bar{x}_n) + w_i, \quad w_i \neq 0$$

for all $i = 1, 2, \ldots, n$.

We note by Lemma 2.1 that W is an acute cone. By the acute property of W and Lemma 2.1, we have $p^{i^T} w_i < 0$, hence

$$p^{i^T} A^i(\bar{x}) < p^{i^T} A^i(\bar{x}_1, \ldots, \bar{x}_{i-1}, x_i, \bar{x}_{i+1} \ldots, \bar{x}_n) \text{ for all } i = 1, \ldots, n.$$

This contradicts (3.1).

Q.E.D.

Lemma 3.3: Let $\Gamma = \left\{\Re_K,\ W;\ A^1, \ldots, A^n\right\}$ be a (W-K)-concave game. If $\bar{x} \in \Re_K$ is a nondominated equilibrium point of Γ associated with W, then there exists strictly nonzero $\bar{p} \in W_n^*$ such that

$$\Psi_{\bar{p}}(x, \bar{x}) \le \Psi_{\bar{p}}(\bar{x}, \bar{x}) \qquad \text{for all } x \in D(\bar{x})$$

Proof: By Lemma 3.1, there exists a strictly nonzero $\bar{p} \in W_n^*$ such that for arbitrary $x \in D(\bar{x})$, we have

$$\bar{p}^{i^T} A^i(\bar{x}_1, \ldots, \bar{x}_{i-1}, x_i, \bar{x}_{i+1} \ldots, \bar{x}_n) \le \bar{p}^{i^T} A^i(\bar{x})$$

for all $i = 1, 2, \ldots, n$, then

$$\sum \bar{p}^{i^T} A^i(\bar{x}_1, \ldots, \bar{x}_{i-1}, x_i, \bar{x}_{i+1} \ldots, \bar{x}_n) \le \sum_{i=1}^{n} \bar{p}^{i^T} A^i(\bar{x})$$

i.e., $$\Psi_{\bar{p}}(x, \bar{x}) \le \Psi_{\bar{p}}(\bar{x}, \bar{x}) \qquad \text{for all } x \in D(\bar{x})$$

Q.E.D.

Lemma 3.4: Let $\Gamma = \left\{\Re_K,\ W;\ A^1, \ldots, A^n\right\}$., $\bar{x} \in \Re_K$ and $A^i(x)$ be differentiable at $\bar{x}$ with respect to x_i for fixed $\bar{x}_1, \ldots, \bar{x}_{i-1}, \bar{x}_{i+1} \ldots, \bar{x}_n$ $(i = 1, \ldots, n)$. If there exists $\bar{p} \in W_n^*$ such that

$$\Psi_{\bar{p}}(x, \bar{x}) \le \Psi_{\bar{p}}(\bar{x}, \bar{x}) \qquad \text{for all } x \in D(\bar{x})$$

then

$$\left(\nabla_{x_i} \Psi_{\bar{p}}(x, \bar{x})\Big|_{x=\bar{x}}\right)^T \in T^*(D_i(\bar{x}), \bar{x}_i) \quad (i = 1, \ldots, n)$$

Proof: For arbitrary $h \in T(D_i(\bar{x}), \bar{x}_i)$, there exists $\left\{x_i^k\right\} \subset D_i(\bar{x})$ with $\lim_{k\to\infty} x_i^k = \bar{x}_i$, and $\lambda_k > 0$ with $\lambda_k \to 0$ such that

$$h = \lim_{k\to\infty} \lambda_k\left(x_i^k - \bar{x}_i\right)$$

since $\Psi_{\bar{p}}(x, \bar{x})$ is differentiable at $\bar{x}$, we have

$$\Psi_{\bar{p}}\left(x^k, \bar{x}\right) = \Psi_{\bar{p}}\left(x^k, \bar{x}\right) + \nabla_{x_i} \Psi_{\bar{p}}\left(x^k, \bar{x}\right)\Big|_{x=\bar{x}}\left(x_i^k - \bar{x}_i\right) + \left\|x_i^k - \bar{x}_i\right\| \cdot \varepsilon(k)$$

where $x^k = \left(\bar{x}_1, \ldots, \bar{x}_{i-1}, x_i^k, \bar{x}_{i+1} \ldots, \bar{x}_n\right)$, $\varepsilon(k) \to 0$ as $k \to \infty$.

Since $\Psi_{\bar{p}}(\bar{x}, \bar{x}) \geq \Psi_{\bar{p}}(x, \bar{x})$ for all $x \in \Re_K$, we have

$$\nabla_{x_i} \Psi_{\bar{p}}(x, \bar{x})\Big|_{x=\bar{x}} \left(x_i^k - \bar{x}_i\right) + \left\|x_i^k - \bar{x}_i\right\| \bullet \varepsilon(k) \leq 0$$

then

$$\nabla_{x_i} \Psi_{\bar{p}}(x, \bar{x})\Big|_{x=\bar{x}} \lambda_k \left(x_i^k - \bar{x}_i\right) + \left\|\lambda_k\left(x_i^k - \bar{x}_i\right)\right\| \bullet \varepsilon(k) \leq 0$$

Letting $k \to \infty$, we have

$$\nabla_{x_i} \Psi_{\bar{p}}(x, \bar{x})\Big|_{x=\bar{x}} h \leq 0$$

Hence

$$\left(\nabla_{x_i} \Psi_{\bar{p}}(x, \bar{x})\Big|_{x=\bar{x}}\right)^T \in T^*(D_i(\bar{x}), \bar{x}_i)$$

Q.E.D

<u>Lemma 3.5</u>: Let $\Gamma = \left\{\Re_K,\ W;\ A^1, \ldots, A^n\right\}$ be a (W-K)-concave game and $\bar{x} \in \Re_K$. For all $i = 1, 2, \ldots, n$, let $A^i(x)$ be differentiable at $\bar{x}$ with respect to x_i for fixed $\bar{x}_1, \ldots, \bar{x}_{i-1}, x_i, \bar{x}_{i+1} \ldots, \bar{x}_n$ $(i = 1, \ldots, n)$. If there exists $\bar{p} \in W_n^*$ such that $\bar{x}$ satisfying

$$\left(\nabla_{x_i} \Psi_{\bar{p}}(x, \bar{x})\Big|_{x=\bar{x}}\right)(y - \bar{x}) \leq 0 \quad \text{for all } y \in D(\bar{x}),$$

then $\bar{x}$ is a nondominated equilibrium point of Γ associated with W.

<u>Proof</u>: By Lemma 3.2, we only need to show that for all $x \in D(\bar{x})$, we have

$$\bar{p}^{i^T} A^i\left(\bar{x}_1, \ldots, \bar{x}_{i-1}, x_i, \bar{x}_{i+1} \ldots, \bar{x}_n\right) \leq \bar{p}^{i^T} A^i(\bar{x}),\ i = 1, \ldots, n.$$

Suppose to the contrary that there exist $\bar{y} \in D(\bar{x})$ and some i_0 $(i \leq i_0 \leq n)$ such that

$$\bar{p}^{i_0^T} A^{i_0}\left(\bar{x}_1, \ldots, \bar{x}_{i_0-1}, x_{i_0}, \bar{x}_{i_0+1} \ldots, \bar{x}_n\right) > \bar{p}^{i_0^T} A^{i_0}(\bar{x}). \tag{3.2}$$

Since Γ is a (W-K)-concave game, by Lemma 2.3, $\bar{p}^{i_0^T} A^{i_0}\left(\bar{x}_1, \ldots, \bar{x}_{i_0-1}, x_{i_0}, \bar{x}_{i_0+1} \ldots, \bar{x}_n\right)$ is concave with respect to x_i, and thus

$$\bar{p}^{i_0^T} A^{i_0}\left(\bar{x}_1, \ldots, \bar{x}_{i_0-1}, x_{i_0}, \bar{x}_{i_0+1} \ldots, \bar{x}_n\right) \leq \bar{p}^{i_0^T} A^{i_0}(\bar{x}) + \bar{p}^{i_0^T} \nabla_{x_{i_0}} A^{i_0}(\bar{x})\left(\bar{y}_{i_0} - \bar{x}_{i_0}\right) \tag{3.3}$$

Since

$$\nabla_x \Psi_{\bar{p}}(x, \bar{x})\Big|_{x=\bar{x}} (y - \bar{x}) \leq 0 \quad \text{for all } y \in D(\bar{x}),$$

Letting $y = \left(\bar{x}_1, \ldots, \bar{x}_{i_0-1}, x_{i_0}, \bar{x}_{i_0+1} \ldots, \bar{x}_n\right)$, we have

$$\nabla_{x_{i_0}} \Psi_{\bar{p}}(x, \bar{x})\Big|_{x=\bar{x}} \left(\bar{y}_{i_0} - \bar{x}_{i_0}\right) \le 0$$

i.e., $$\bar{p}^{i_0^T} \nabla x_{i_0} A^{i_0}(\bar{x}) \left(\bar{y}_{i_0} - \bar{x}_{i_0}\right) \le 0$$

By (3.3), we have

$$\bar{p}^{i_0^T} A^{i_0}\left(\bar{x}_1, \dots, \bar{x}_{i_0-1}, x_{i_0}, \bar{x}_{i_0+1}, \dots, \bar{x}_n\right) \le \bar{p}^{i_0^T} A^{i_0}(\bar{x})$$

This contradicts (3.2).

Q.E.D

Assuming $\bar{x} \in \Re_K$ let

$$c_i(\bar{x}) = \left\{ \left(\nabla_{x_i} G(\bar{x})\right)^T : q \in (-K^*) \text{ such that } q^T G(\bar{x}) = 0 \right\}$$

Lemma 3.6[1] Let $\bar{x} \in \Re_K$ and $G(x)$ be differentiable at $\bar{x}$.

Then

$$T(D_i(\bar{x}), \bar{x}_i) \subset (-C^*_i(\bar{x}))$$

Definition 3.1: A point $\bar{x} \in \Re_K$ is said to be a "generalized regular point" of the constraint set $\Re_K$ if

$$T^*(D_i(\bar{x}), \bar{x}_i) \subset (-C^*_i(\bar{x})).$$

Definition 3.2: If there exist

$\bar{x} = \bar{x}_1, \dots, \bar{x}_n \in E^{n_1} x \dots x E^{n_n}$, $\bar{Q} \in -K^*_n$ and strictly nonzero $\bar{p} \in W^*_n$ such that

$$\nabla_{x_i} \Phi_{\bar{p}}\left(x, \bar{x}, \bar{Q}\right)\Big|_{x=\bar{x}} = \bar{p}^{i^T} \nabla_{x_i} A^i(\bar{x}) + \bar{q}^{i^T} \nabla_{x_i} G(\bar{x}) = 0, \ \bar{x}_i \in S_i \tag{3.4}$$

$$\nabla_{q_i} \Phi_{\bar{p}}\left(x, \bar{x}, \bar{Q}\right) = G(\bar{x}) \in K \tag{3.5}$$

$$\bar{q}^{i^T} \nabla_{q_i} \Phi_{\bar{p}}\left(x, \bar{x}, \bar{Q}\right) = \bar{q}^{i^T} G(\bar{x}) = 0 \tag{3.6}$$

for all i = 1, ..., n, then we say that $(\bar{x}, \bar{p}, \bar{Q})$ satisfies the "generalized Kuhn Tucker" conditions.

Theorem 3.1: Let $\Gamma = \left\{\Re_K, \ W; A^1, \dots, A^n\right\}$ be a (W-K)-concave game. Furthermore, suppose that $\bar{x} \in \Re_K$ is a nondominated equilibrium point of Γ associated with W, and that $\bar{x}$ is a "generalized regular point" of $\Re_K$. Then there exist $\bar{Q} \in -K^*_n$ and strictly nonzero $\bar{p} \in W^*_n$ such that $(\bar{x}, \bar{p}, \bar{Q})$ satisfies the generalized Kuhn - Tucker conditions (3.4) - (3.6).

Proof: By Lemma 3.3, there exists a strictly nonzero $\bar{p} \in W^*_n$ such that

$$\Psi_{\bar{p}}(x, \bar{x}) \le \Psi_{\bar{p}}(\bar{x}, \bar{x}) \qquad \text{for all } x \in D(\bar{x})$$

Since $\bar{x}$ is a generalized regular point of $\Re_K$ and

$$\left(\nabla_{x_i}\Psi_{\bar{p}}(x,\bar{x})\Big|_{x=\bar{x}}\right)^T \in T^*(D(\bar{x}),\bar{x}_i) \quad \text{(see Lemma 3.4)},$$

there exist $\bar{q}^i \in E^m$ such that

$$\nabla_{x_i}\Psi_{\bar{p}}(x,\bar{x})\Big|_{x=\bar{x}} + \bar{q}^{i^T}\nabla_{x_i}G(\bar{x}) = 0$$

and

$$\bar{q}^i \in (-K^*),\ \bar{q}^{i^T}G(\bar{x}) = 0$$

for all i = 1, 2, ..., n.

i.e.,

$$\nabla_{x_i}\Phi_{\bar{p}}(x,\bar{x},\bar{Q})\Big|_{x=\bar{x}} = \bar{p}^{i^T}\nabla_{x_i}A^i(\bar{x}) + \bar{q}^{i^T}\nabla_{x_i}G(\bar{x}) = 0,\ \bar{x}_i \in S_i$$

$$\nabla_{q_i}\Phi_{\bar{p}}(\bar{x},\bar{x},\bar{Q}) = G(\bar{x}) \in K$$

$$\bar{q}^{i^T}\nabla_{q_i}\Phi_{\bar{p}}(\bar{x},\bar{x},\bar{Q}) = \bar{q}^{i^T}G(\bar{x}) = 0$$

for all i = 1, 2, ..., n. where $\bar{Q} = (\bar{q}^1, \ldots, \bar{q}^n) \in -K_n^*$

Q.E.D.

<u>Theorem 3.2</u>: Let $\Gamma = \{\Re_K,\ W;\ A^1, \ldots, A^n\}$ be a (W-K)-concave game, let K be closed and $\bar{x} \in \Re_K$, let A^i be differentiable at $\bar{x}$ with respect to x_i for fixed $\bar{x}_1, \ldots, \bar{x}_{i-1}, \bar{x}_{i+1}\bar{x}_n$ and G(x) be differentiable at $\bar{x}$. If there exist $\bar{p} \in \operatorname{Int} W_n^*$ and $\bar{Q} \in -K_n^*$ such that $(\bar{x},\bar{p},\bar{Q})$ satisfies the generalized Kuhn-Tucker conditions (3.4) - (3.6), then $\bar{x}$ *is a nondominated equilibrium point of* Γ associated with W.

<u>Proof</u>: According to Lemma 3.5, it suffices to show that

$$\nabla_x\Psi_{\bar{p}}(x,\bar{x})\Big|_{x=\bar{x}}(y-\bar{x}) \le 0 \quad \text{for all } y \in D(\bar{x}),$$

For arbitrary $y \in \Re_K$, we have by Lemma 2.2,

$$G(\bar{x}) + \nabla_x G(\bar{x})(y-\bar{x}) \in G(y) + K$$

Hence

$$\bar{q}^{i^T}\Delta_x G(\bar{x})(y-\bar{x}) \ge 0 \quad \text{for any } y \in \Re_K$$

letting $y = (\bar{x}_1, \ldots, \bar{x}_{i-1}, y_i, \bar{x}_{i+1}, \ldots, \bar{x}_n) \in \Re_K$, then

$$\bar{q}^{i^T}\Delta_{x_i}G(\bar{x})(y-\bar{x}) \ge 0 \quad \text{for all } y_i \in D_i(\bar{x})$$

and then

$$\nabla_{x_i}\Psi_{\bar{p}}(x,\bar{x})\big|_{x=\bar{x}}(y_i-\bar{x}_i) = q^{i^T}\nabla_{x_i}G(\bar{x})(y_i-\bar{x}_i) \le 0$$

for all $y_i \in D_i(\bar{x})$, $i = 1, 2, \ldots, n$;

i.e.,

$$\nabla_x\Psi_{\bar{p}}(x,\bar{x})\big|_{x=\bar{x}}(y-\bar{x}) \le 0 \qquad \text{for all } y \in D(\bar{x})$$

Q.E.D.

Now, let's consider the example in Harker's paper, the two -person game depicted in Fig. 1. Each player chooses a number x_i between 0 and 10, such that the sum of these numbers is less than or equal to 15. Harker's utility functions and constraint functions are defined in our terms by:

$$A_1(x_1,x_2) = 34x_1 - x_1^2 - (8/3)\,x_1x_2$$

$$A_2(x_1,x_2) = 24.25x_2 - x_2^2 - (5/4)\,x_1x_2$$

$$G(x_1,x_2) = \begin{pmatrix}10-x_1, & x_1, & 10-x_2, & x_2, & 15-x_1-x_2\end{pmatrix}^T$$

Here $W = E^1_-$, $K = E^5_+$, $S = E^2$, $\Re_K = \left\{x = (x_1,x_2)^T : G(x_1,x_2) \in K\right\}$

since G(x) is linear, for any point $x \in \Re_K$, x is a *generalized regualr point* of $\Re_K$. *It is easy to check t. at* $\Gamma = \left\{\Re_K,\ W;\ A_1, A_2\right\}$ is a (W - K)-concave game. Hence by Theorem 3.1 and Theorem 3.2, these nondominated equilibrium points of Γ are points which satisfy the generalized Kuhn-Tucker conditio ›s (3.4) - (3.6) and vice-versa.

Let
$$\Phi(x,y,\lambda) = A_1(x_1,y_2) + A_2(y_1,x_2) + \lambda^{1^T}G(x_1,y_2) + \lambda^{2^T}G(y_1,x_2)$$

where $\lambda = (\lambda^1,\lambda^2)$, $\lambda^1 = \left(\lambda_1^1,\ldots,\lambda_5^1\right)^T$, $\lambda^2 = \left(\lambda_1^2,\ldots,\lambda_5^2\right)^T$.

Then the generalized Kuhn-Tucker conditions of Γ are to find $\left(\bar{x},\bar{\lambda}^1,\bar{\lambda}^2\right)$ satisfying:

$$\nabla_{x_1}\Phi(x,\bar{x},\bar{\lambda})\big|_{x=\bar{x}} = 34 - 2\bar{x}_1 - (8/3)\bar{x}_2 - \lambda_1^1 + \lambda_2^1 - \lambda_5^1 = 0$$

$$\nabla_{x_2}\Phi(x,\bar{x},\bar{\lambda})\big|_{x=\bar{x}} = 24.25 - 2\bar{x}_2 - (5/4)\bar{x}_1 - \lambda_3^2 + \lambda_4^2 - \lambda_5^2 = 0$$

$$\nabla_{\lambda^i}\Phi(\bar{x},\bar{x},\bar{\lambda}) = \left(10-\bar{x}_1, \bar{x}_1, 10-\bar{x}_2, \bar{x}_2, 15-\bar{x}_1, \bar{x}_2\right)^T \ge 0,\ \lambda^i \ge 0,\ i = 1, 2.$$

$$\bar{\lambda}^{i^T}\nabla_{\lambda^i}\Phi(\bar{x},\bar{x},\bar{\lambda}) = \bar{\lambda}_1^i(10-\bar{x}_1) + \bar{\lambda}_2^i\bar{x}_1 + \bar{\lambda}_3^i(10-\bar{x}_2) + \bar{\lambda}_4^i\bar{x}_2 + \bar{\lambda}_5^i(15-\bar{x}_1-\bar{x}_2) = 0$$

Simplifying the above, this is equivalent to finding $\left(\bar{x},\bar{\lambda}^1,\bar{\lambda}^2\right)$ satisfying the following system:

$$34 - 2x_1 - (8/3)x_2 = \lambda_1^1 - \lambda_2^1 + \lambda_5^1 \tag{3.7}$$

$$24.25 - 2x_2 - (5/4)x_1 = \lambda_3^2 - \lambda_4^2 + \lambda_5^2 \tag{3.8}$$

$$\lambda_1^i(10 - x_1) = 0 \ , \ x_1 \le 10, \lambda_1^i \ge 0 \ , \ i = 1, 2 \tag{3.9}$$

$$\lambda_2^i x_1 = 0 \ , \ x_1 \ge 0, \lambda_2^i \ge 0 \ , \ i = 1, 2 \tag{3.10}$$

$$\lambda_3^i(10 - x_2) = 0 \ , \ x_2 \le 10, \lambda_3^i \ge 0 \ , \ i = 1, 2 \tag{3.11}$$

$$\lambda_4^i x_2 = 0 \ , \ x_2 \ge 0, \lambda_4^i \ge 0 \ , \ i = 1, 2 \tag{3.12}$$

$$\lambda_5^i(15 - x_1 - x_2) = 0 \ , \ \lambda_5^i \ge 0 \ , \ x_1 + x_2 \le 15 \tag{3.13}$$

The constraint set $\mathfrak{R}_K$ is illustrated in Fig. 1.

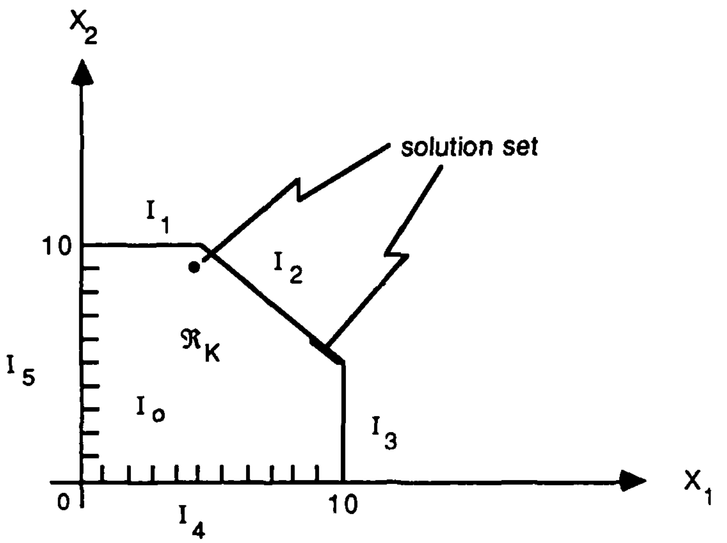

Fig. 1.

We divide $\mathfrak{R}_K$ into six parts I_i , $i = 0, 1, 2, 3, 4, 5$:

$I_0 = \text{Int}\,\mathfrak{R}_K = \left\{(x_1, x_2) : 0 < x_1 < 10, 0 < x_2 < 10, \ x_1 + x_2 < 15\right\}$

$I_1 = \left\{(x_1, x_2) : 0 \le x_1 \le 5, x_2 = 10\right\}$

$I_2 = \left\{(x_1, x_2) : 5 < x_1 \le 10, \ x_1 + x_2 = 15\right\}$

$I_3 = \left\{(x_1, x_2) : x_1 = 0, 0 < x_2 < 5\right\}$

$I_4 = \left\{(x_1, x_2) : 0 < x_1 \le 10, x_2 = 0\right\}$

$I_5 = \left\{(x_1, x_2) : x_1 = 0, 0 \le x_2 < 10\right\}$, For each part I_i, using the complementary conditions (3.9) - (3.13), it is easy to figure out which points satisfy the system (3.7) - (3.13). The process is the following:

(1) I_o: Since $0 < x_1 < 10$, $0 < x_2 < 10$ and $x_1 + x_2 < 15$, by (3.9) - (3.13) we have $\lambda_j^i = 0$ $i = 1, 2,$ $j = 1, 2, 3, 4, 5$. By (3.7) and (3.8), we have

$$34 - 2x_1 - (8/3)x_2 = 0$$

$$24.25 - 2x_2 - (5/4)x_1 = 0$$

then $(\bar{x}_1, \bar{x}_2) = \underline{(5.9)}$ satisfies (3.7) - (3.13)

(2) I_1: Since $0 \le x_1 \le 5$ and $x_2 = 10$, by (3.9) and (3.12) we have $\lambda_1^i = 0$, $\lambda_4^i = 0$.

By (3.7) and (3.8), we have

$$34 - 2x_1 - (80/3) = -\lambda_2^1 + \lambda_5^1 \tag{3.14}$$

$$24.25 - 20 - (5/4)x_1 = \lambda_3^2 + \lambda_5^2 \tag{3.15}$$

(a) $x_1 = 0$. By (3.13) we have $\lambda_5^i = 0$, $i = 1, 2$.

By (3.14), $\lambda_2^i = -(22/3) < 0$, which contradicts (3.10).

(b) $x_1 = 5$. by (3.10) we have $\lambda_2^i = 0$, $i = 1, 2$.

By (3.14), $\lambda_5^i = -(8/3) < 0$, which contradicts (3.13).

(c) $0 < x_1 < 5$. By (3.10) and (3.13) we have $\lambda_2^i = \lambda_5^i = 0$ $(i = 1, 2)$ and then by (3.14) and (3.15), we have

$x_1 = 11/3$

$\lambda_3^2 = -1/3 < 0$, which contradticts (3.11).

(3) I_2: Since $5 < x_1 \le 10$ and $x_1 + x_2 = 15$ (obviously $0 < x_2 = 15 - x_1 < 10$), by (3.10) - (3.12) we have

$\lambda_2^i = \lambda_3^i = \lambda_4^i = 0$, $i = 1, 2$.

then by (3.7) and (3.8), we have

$$34 - 2x_1 - (8/3)(15 - x_1) = \lambda_1^1 + \lambda_5^1$$

$$24.25 - 2(15 - x_1) - (5/4)x_1 = \lambda_5^2$$

i.e.,

$$\lambda_5^1 = (2/3)x_1 - 6 - \lambda_1^1 \tag{3.16}$$

$$\lambda_5^2 = (3/4)x_1 - 5.75 \tag{3.17}$$

(a) $x_1 = 10$. By (3.16) and (3.17) , we have

$$\lambda_1^1 + \lambda_5^1 = 2/3 \tag{3.18}$$

$$\lambda_5^2 = 7/4 \tag{3.19}$$

so we can find $\lambda_1^1 \geq 0$, $\lambda_5^1 \geq 0$, $\lambda_5^2 = 7/4$ to satisfy (3.18) and (3.19), i.e., $\underline{(10, 5)}$ satisfies the generalized Kuhn - Tucker conditions.

(b) $x_1 < 10$. By (3.9) we have $\lambda_1^i = 0$ $i = 1, 2$. Then by (3.16) and (3.17), we have

$$\lambda_5^1 = (2/3)x_1 - 6$$

$$\lambda_5^2 = (3/4)x_1 - 5.75$$

letting $\lambda_5^1 \geq 0$, $\lambda_5^2 \geq 0$, we have

$$x_1 \geq 9 \text{ and } x_1 \geq 23/3$$

but $x_1 \geq 9$ implies $x_1 \geq 23/3$.

Combining (a) and (b), the segment

$$I_1^2 = \{(x_1, x_2): 9 \leq x_1 \leq 10, x_2 = 15 - x_1\}$$

is the solution set of system (3.7) - (3.13) in I_2, i.e., that satisfying the generalized Kuhn - Tucker conditions.

(4) I_3: Since $x_1 = 10$, $0 < x_2 < 5$, by (3.10) - (3.13) we have

$$\lambda_2^i = \lambda_3^i = \lambda_4^i = \lambda_5^i = 0 \quad , \quad i = 1, 2$$

By (3.7) and (3.8) , we have

$$34 - 20 - (8/3)x_2 = \lambda_1^1$$

$$24.25 - 2x_2 - 50/4 = 0$$

then

$$x_2 = 47/8$$

$\lambda_1^1 = -5/3 < 0$, which contradicts (3.9).

(5) I_4: Since $0 < x_1 \leq 10$, $x_2 = 0$, by (3.10), (3.11) and (3.13) we have

$$\lambda_2^i = \lambda_3^i = \lambda_5^i = 0 \quad , \quad i = 1, 2$$

By (3.7) and (3.8) , we have

$$34 - 2x_1 = \lambda_1^1$$

$$24.25 - (5/4)x_1 = -\lambda_4^2$$

Then $\lambda_4^2 = (5x_1 - 97)/4 < 0$ which contradicts (3.12).

(6) I_5: Since $x_1 = 0$ and $0 \le x_2 < 10$, by (3.9), (3.11) and (3.13), we have

$$\lambda_1^i = \lambda_3^i = \lambda_5^i = 0 \quad , \quad i = 1, 2$$

By (3.7) and (3.8), we have

$$34 - (8/3)x_2 = -\lambda_2^1$$

$$24.25 - 2x_2 = -\lambda_4^2$$

Then $\lambda_2^1 = (8x_2 - 102)/3 < 0$ which contradicts (3.10).

Hence combining (1) - (6), the solution set of system (3.7) - (3.13) is composed of the point (5,9) and the interval [(9,6) , (10,5)].

By Theorem 3.1 and 3.2, the set of nondominated equilibrium points of Γ associated with W is composed of the point (5,9) and the interval [(9,6), (10,5)] (see fig. 1).

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Models employed in evaluation or stipulation of regulatory policies involving competition and/or differing objectives of competing parties have sometimes been seriously deficient in accounting for interactions between the parties' strategies or in allowing for multiple objectives. Classical game-theoretical models, wherein the strategy set is the topological product of the individual parties strategy sets, do not encompass such situations. The new "dominance cones" method and class of solutions (Charnes, Cooper, Huang and Wei) is herein further extended to such extensions of classical games. This is

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) applied to an example of Harker without requiring his variational and quasi-variational inequalities or point-to-set mappings.